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# COMMENTS ON A PENETRATION THEORY FOR AN UNDEFORMED PROJECTILE BY AWERBUCH AND BODNER

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**Abstract**—This paper deals with a discussion on some important aspects of a theory of plate perforation by projectile developed by Awerbuch and Bodner. The axial expansion velocity of the combined projectile, $U$, has been differentiated from the projectile particle velocity, $V$. The expressions of the inertial force and effective mass of the combined projectile, and the appropriate equation for the first two stages have been derived.

## INTRODUCTION

Awerbuch and Bodner[1] developed a theory of plate perforation that considers target deformation in three interconnected stages. This theory has been compared with a series of experiments[2], and the prediction is consistent with the data. However, the formulation of the equation of motion in the first and second stages of the penetration has some questionable terms; the longitudinal expansion velocity of the plug, $U$, differs from that of a projectile in both concept and magnitude. If the relationship satisfied $U \leq V$, the plugging process cannot occur. It is stated that the relationship between velocity $U$ and $V$ just corresponds to that between the disturbance wave and a particle behind a wave front. In the theory, Awerbuch and Bodner equate the axial expansion velocity of the plug to that of the moving projectile. Tate[3] has commented on the concept of additional mass, and derived the equation of momentum for the first stage by means of the theory of potential flow. The present paper discusses the two kinds of velocity, gives the equation of motion of the combined projectile in the first and second stages, and determines the inertial force that acts on the projectile.

## DISCUSSION

The relationship between added mass and penetration distance from the initial surface of the target is shown schematically for the first or second stages (see Fig. 1).

### 1. *Two different kinds of velocity*

The penetration depth of the projectile is denoted by $x$ in the text. It is important to note that $x$ is the distance from the front of the added mass and not the penetration of the original projectile [see [1], Figs. 1(a)–1(c)]. The plug is formed by the increase in length due to the added mass; the length of added mass cannot remain constant during penetration. At any instantaneous time $t$, the distance of the projectile nose from the initial surface of the target is $s$ (see Fig. 1), the longitudinal length of added mass becomes

$$\delta = x - s. \tag{1}$$

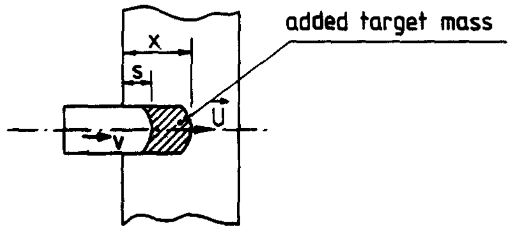


Fig. 1. Schematic of added target mass due to longitudinal expansion.

The rate of increase in length of added mass is

$$\frac{d\delta}{dt} = \frac{dx}{dt} - \frac{ds}{dt}. \tag{2}$$

Let $dx/dt = U$ be the axial expansion velocity of the combined projectile, and let the projectile particle velocity be $V$. The velocity of the original projectile nose, $ds/dt$, is just $V$ because the projectile's nose has become an ordinary point in the combined projectile. The relationship $d\delta/dt$ becomes rate of change in longitudinal length of added mass. Equation (2) may be rewritten

$$\frac{d\delta}{dt} = U - V. \tag{3}$$

The relationship between the velocities $U$ and $V$ resembles that between the velocities of plastic wave and particle behind the wave front. The velocities differ in both concept and magnitude.

When

$$U > V; \quad \frac{d\delta}{dt} > 0,$$

the part of the target material which is compressed joins to the projectile at velocity $U$, and then moves at the same velocity $V$. The longitudinal length of added mass increases at the rate $d\delta/dt$. The second stage ends when the compressed plug encompasses the entire target thickness. This is joined to the projectile and both move at the same velocity.

When

$$U \leq V; \quad \frac{d\delta}{dt} \leq 0,$$

the gross rate of compressible deformation of the target material exceeds the capacity of the material to propagate deformation. Consequently the plastic deformation cannot get away from the interface and erosion of the target material occurs. Nevertheless, [1] assumes $U = V$. This leads to some terms not appropriate to the inertial force and effective mass of combined projectile.

2. *Expression of inertial force*

According to Awerbuch and Bodner's analysis, by equating the work done by the reaction of the inertial force on the target material to the change in the kinetic energy of the displaced material, the inertial force is obtained. The mass element $dm$ of the target material, the increment of the additional mass, which is displaced by the projectile as it advanced $dx_n$ would be

$$dm = \rho\, dx_n\, dA_n. \tag{4}$$

The composite projectile advances a corresponding distance $ds$. Assuming the inertial force $dF_{in}$ acts on the elemental area $dA_n$, this leads to

$$dF_{in}\, ds = \tfrac{1}{2}\, dm V_n^2, \tag{5}$$

or

$$dF_{in} = \tfrac{1}{2}\rho\, (dA_n) \frac{dx_n}{ds} V_n^2. \tag{6}$$

Note that

$$\frac{dx_n}{dt} = \frac{dx_n}{dt} \cdot \frac{dt}{ds} = \frac{U}{V_n}, \tag{7}$$

so

$$dF_{in} = \tfrac{1}{2}\rho\, (dA_n) U V_n. \tag{8}$$

For various shapes of the projectile nose, the ordinary form of the inertial force $F_i$ gives

$$F_i = \tfrac{1}{2}\kappa \rho A U V. \tag{9}$$

Equation (9) differs from eqn (5) in [1], in that $V^2$ is replaced by $UV$. Velocity $U$ is really that of the propagation of large disturbances for a plastic wave. In the text, neglecting the deformation and mass loss of projectile, the target material suffers elastic and plastic deformation since the impact velocity is not large. From the point of view of the wave motion, eqn (5), which appears to be hydrodynamic form in the original analysis, is correct only when $V \geq U$. In this situation, erosion occurs rather than the plugging process, as described in [1].

3. *Effective mass of combined projectile*

According to the above analysis, the eqns (7), (11) and (8) in [1], which have been substituted into the relationship $dx/dt = V$, are not appropriate. The appropriate forms are

$$\frac{dm}{dt} = \rho A \frac{dx}{dt} = \rho A U \tag{10}$$

and

$$\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = U \frac{dV}{dx}. \tag{11}$$

4. *Equation of motion in the first or second stages*

In [1], inertial force acts on the projectile's nose and does not act on the front of the additional mass. The equation of motion of the projectile is

$$m_0 \frac{dV}{dt} = -(F_i + F_c + F_s). \tag{12}$$

For the combined projectile

$$\frac{d}{dt}(mV) = -(F_i' + F_c + F_s), \tag{13}$$

where the inertial force $F_i'$, which acts on the front of the additional mass, differs from the inertial force $F_i$. Because additional mass would generate an additional inertial force $\Delta F_i$ and this in turn decreases $F_i$,

$$F_i' = F_i - \Delta F_i, \tag{14}$$

from the conservation of momentum,

$$\Delta F_i \, \mathrm{d}t = \mathrm{d}mV \tag{15}$$

so that

$$\Delta F_i = V \frac{\mathrm{d}m}{\mathrm{d}t}. \tag{16}$$

Substituting eqn (16) into eqn (14) gives

$$F_i' = F_i - V \frac{\mathrm{d}m}{\mathrm{d}t}. \tag{17}$$

Putting the eqn (17) into the equation of motion (13),leads to

$$m \frac{\mathrm{d}V}{\mathrm{d}t} + V \frac{\mathrm{d}m}{\mathrm{d}t} = -(F_i - V \frac{\mathrm{d}m}{\mathrm{d}t} + F_c + F_s), \tag{18}$$

so that

$$m \frac{\mathrm{d}V}{\mathrm{d}t} = -(F_i + F_c + F_s). \tag{19}$$

Equation (19) is an appropriate form of the equation of motion in the second stage.

In addition eqn (19) could alternatively be obtained by means of the method of a control volume. Taking the instantaneous combined projectile as a control volume, the momentum would be

$$M = (m_0 + \rho A x)V = mV. \tag{20}$$

The momentum rate of the outflow control volume would be

$$\Phi = -\rho A \frac{\mathrm{d}x}{\mathrm{d}t} V, \tag{21}$$

where the symbol "$-$" represents momentum flowing into the control volume. The resultant force acting on the control volume is

$$F = -(F_i + F_c + F_s). \tag{22}$$

It is noted that the inertial force $F_i$, which also acts on the front of the combined projectile, differs from $F_i'$ in eqn (14) in this case. Substituting eqns (20)–(22) into the following equation of momentum,

$$\frac{\mathrm{d}}{\mathrm{d}t} M + \Phi = -(F_i + F_c + F_s), \tag{23}$$

leads to

$$m \frac{\mathrm{d}v}{\mathrm{d}t} + V \frac{\mathrm{d}m}{\mathrm{d}t} - \rho A \frac{\mathrm{d}x}{\mathrm{d}t} V = -(F_i + F_c + F_s). \tag{24}$$

Note that

$$\frac{dm}{dt} = \rho A \frac{dx}{dt}.$$

Subtituting the above equation into eqn (24) gives

$$m \frac{dV}{dt} = -(F_i + F_c + F_s),$$

which is the same as eqn (19).

It seems that the term, $V\, dm/dt$, may always be cancelled in eqn (10) in the text. Sun Gengchen et al.[4] has obtained the same equation of motion for a similar case using a coordinate transformation. Recht[5] indicates that the equation of motion should be $m\, dV/dt = F$ in this case, and $F = d(mV)/dt$ does not apply [5]. Tate[6] and Lee and Wolf[7] have also given this form of the equation of motion in their respective papers.

According to the above analysis, the equation of motion in the first and second stages in the text presented by Awerbuch and Bodner is not appropriate.

## CONCLUSION

Keeping the basic assumptions of [1], two different kinds of velocity, $U$ and $V$, in the penetration process have been recognized. The expression for effective mass of combined projectile and inertial force has been determined, and the appropriate equation of motion for the first and second stages have been derived.

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